

Toothpick and Staircase Patterns

Reporting Category Patterns, Functions, and Algebra

Topic Working with numerical and geometric patterns

Materials

- Toothpicks
- Centimeter cubes
- Larger cubes for display purposes (optional)
- Building Staircases Recording Sheet (attached)
- Number Patterns Game Directions (attached)
- Number Patterns Game Cards (attached)
- Number Patterns Game Board (attached)

Vocabulary

geometric pattern, numerical pattern, function, rule, growing, repeating, extend

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Distribute piles of toothpicks so students may create the strip pattern shown below. Have students use 4 toothpicks to make the first term in the pattern (a square). Then, have them use 7 toothpicks to make the second term in the pattern (2 connected squares), 10 toothpicks to make the third term in the pattern (3 connected squares), and 13 toothpicks to make the fourth term in the pattern (4 connected squares).



2. Direct students to organize the data in table form, as shown at right, and predict the number of toothpicks in 5 connected squares. Display the same information for all to see.
3. Before students can understand the meaning of the rule, they must discuss what makes the rule. Have them look at the numbers in the table and come to their own understanding about what is happening in the pattern. Then, have them predict the number of toothpicks each time a square is added. Have students explain the pattern. (Note: All fourth graders should not be expected to know the rule, let alone understand it.) Ask, "How can you determine the number of toothpicks required to make five connected squares? What is the rule?" $[(n \times 3) + 1]$
4. Distribute copies of the Building Staircases Recording Sheet. Use centimeter cubes (or larger display cubes) to build a staircase model, beginning with one step (one cube) and continuing until four steps have been created. As you increase the height of the staircase, ask, "In this staircase pattern, which step is this? How many cubes are need to make this step? How many cubes does the staircase pattern have so far? How many cubes will we

Number of squares	Number of toothpicks
1	4
2	7
3	10
4	13
5	

needed to add to make the next step? How many cubes will the pattern have then?" Have students record answers to these questions on their recording sheets.

5. Distribute collections of centimeter cubes, and direct pairs of students to use the cubes to build staircase models. Have students record the patterns they see. Ask, "How many centimeter cubes would you need to build a staircase with one hundred steps?" Help students explore different possibilities for finding the answer.

Assessment

- **Questions**
 - Imagine a cube staircase pattern with six steps. How many centimeter cubes does the pattern have? (Draw the model, if you need to.) How many centimeter cubes would you need to add to make the next step? How many centimeter cubes would the pattern have then?
 - How would you determine the number of centimeter cubes needed to build a staircase with any particular number of steps?
- **Journal/Writing Prompts**
 - Have students glue their first four toothpick terms from step 1 in their journals to keep as a reference. Have them write about the sequence of terms—what changes from one term to the next and what stays the same.
 - Draw a number line, and label on the line the number of toothpicks in the toothpick strip in step 1 (4, 7, 10, 13, 16). Describe how the visual of the number line can help you determine the pattern.
- **Other**
 - Observe students as they build models and generalize patterns.
 - Use the Building Staircases handout for skill assessment.

Extensions and Connections (for all students)

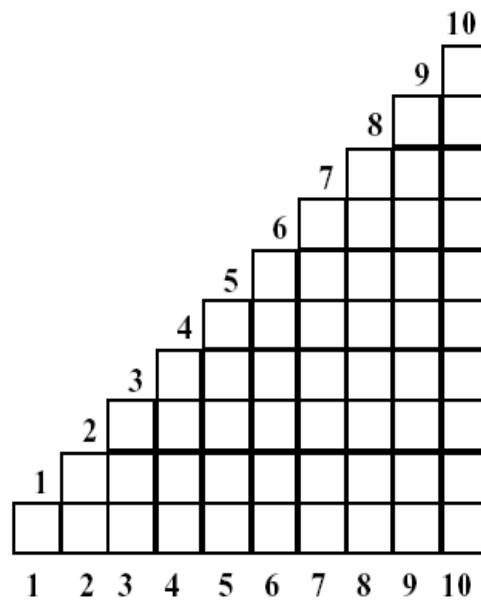
- Have students use a hundred chart to find and create numerical patterns.
- Have students use a number line to create and extend numerical patterns.
- Have students play Number Patterns Game, using the three attachments.

Strategies for Differentiation

- Have students use such items as cotton swabs, pipe cleaners, licorice, or base-10 rods instead of toothpicks.
- Provide visual representations of the shapes for students to refer to in the instructional activities.
- Have students use enlarged numbers and the letter n on cards to create the rules and generalizations in the instructional activities.
- Have students use linking cubes or wooden blocks for building staircases.
- Provide visual representations for the different shapes that are created.
- Have students use a geoboard instead of toothpicks.
- Have students do a "Think, Pair, Share" after completing each activity to share what they have discovered or learned.
- Enlarge the game cards and game board.

Building Staircases Recording Sheet

Name _____ Date _____



Step	Total Number of Cubes
1	
2	
3	
4	
5	
.	
.	
100	
n	

Number Patterns Game Directions

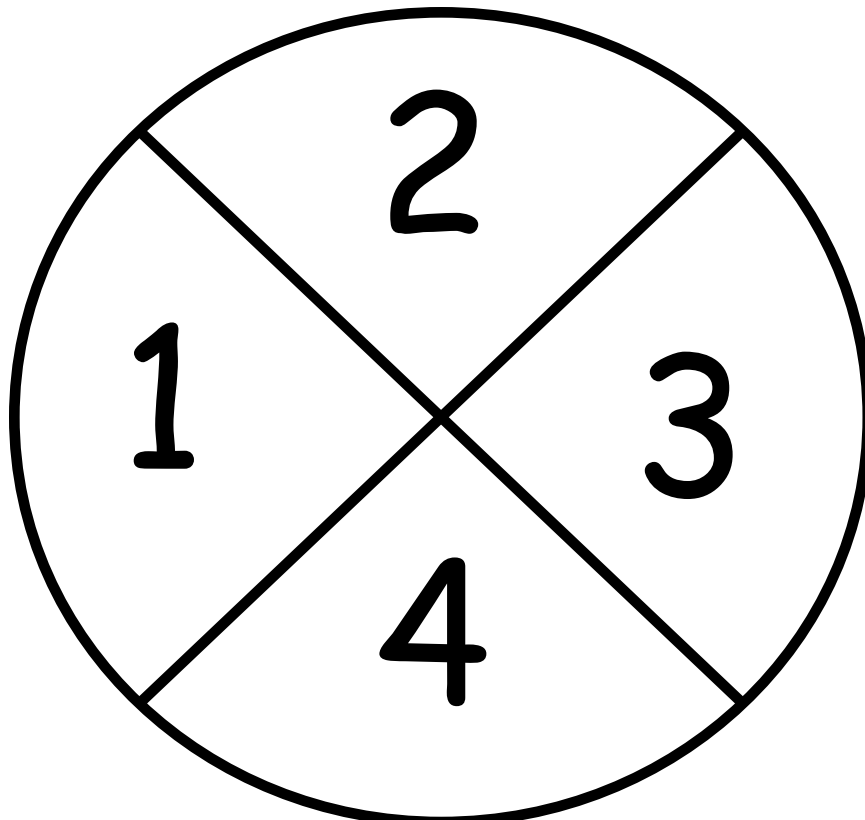
Materials Needed

- Game board
- Game cards
- Game pieces
- Pencil and paper clip
- Scratch paper
- Calculators (optional)

How to Play (2–4 players)

1. Players place the game cards face down on the game board, as indicated.
2. Players place their game pieces at the start line. Each player spins one time, and the player with the largest spin goes first. Play continues clockwise around the group.
3. The first player draws a game card and answers the question. If the answer is correct, he/she spins the spinner and moves his/her game piece that many squares on the board.
4. The next player takes a turn, and the game continues.
5. The player to reach the *FINISH* square first, wins the game.

Spinner



Number Patterns Game Cards

Copy the cards on cardstock, and cut them out.

<p>What is the rule for this pattern?</p> <p>13, 16, 19, 22, 25...</p>	<p>What is the rule for this pattern?</p> <p>42, 40, 38, 36...</p>	<p>What is the rule for this pattern?</p> <p>35, 40, 45, 50...</p>
<p>What is the rule for this pattern?</p> <p>15, 17, 19, 21...</p>	<p>What is the rule for this pattern?</p> <p>19, 16, 13, 10...</p>	<p>What is the rule for this pattern?</p> <p>1, 2, 4, 7, 11, 16...</p>
<p>What is the rule for this pattern?</p> <p>2, 4, 8, 16, 32...</p>	<p>What is the rule for this pattern?</p> <p>1, 5, 25, 125, 625...</p>	<p>What is the rule for this pattern?</p> <p>50, 100, 150, 200...</p>
<p>What is the next number in this pattern?</p> <p>18, 24, 30, 36, __</p>	<p>What is the next number in this pattern?</p> <p>21, 28, 35, 42, __</p>	<p>What is the next number in this pattern?</p> <p>64, 56, 48, 40, __</p>
<p>What is the next number in this pattern?</p> <p>126, 128, 130, 132, __</p>	<p>What is the next number in this pattern?</p> <p>36, 45, 54, 63, __</p>	<p>What is the next number in this pattern?</p> <p>37, 49, 61, __, 85</p>
<p>What is the next number in this pattern?</p> <p>86, 82, __, 74, 70</p>	<p>What is the next number in this pattern?</p> <p>43, 48, 53, __, 63</p>	<p>What is the next number in this pattern?</p> <p>28, 35, __, 49, 56</p>

<p>Finish the pattern</p> <p>Rule: Add 8</p> <p>13, __, __, __</p>	<p>Finish the pattern</p> <p>Rule: Subtract 3</p> <p>86, __, __, __</p>	<p>Finish the pattern</p> <p>Rule: Multiply by 2</p> <p>1, __, __, __</p>
<p>Finish the pattern</p> <p>Rule: Add 16</p> <p>21, __, __, __</p>	<p>Finish the pattern</p> <p>Rule: Multiply by 3</p> <p>1, __, __, __</p>	<p>Finish the pattern</p> <p>Rule: Add 75</p> <p>20, __, __, __</p>
<p>Finish the pattern</p> <p>Rule: Subtract 15</p> <p>90, __, __, __</p>	<p>Finish the pattern</p> <p>Rule: Subtract 9</p> <p>100, __, __, __</p>	<p>Finish the pattern</p> <p>Rule: Add 31</p> <p>77, __, __, __</p>
<p>What are the missing numbers in the pattern?</p> <p>6, 9, __, 15, 18, __, 24, 27</p>	<p>What are the missing numbers in the pattern?</p> <p>1, 2, 4, __, 11, __, 22, 29</p>	<p>What are the missing numbers in the pattern?</p> <p>3, 6, __, 24, __, 96, 192</p>
<p>What are the missing numbers in the pattern?</p> <p>12, 16, __, 24, 28, __, 36</p>	<p>What are the missing numbers in the pattern?</p> <p>27, 36, __, 54, 63, __, 81</p>	<p>What are the missing numbers in the pattern?</p> <p>36, 32, __, 24, 20, __, 12</p>
<p>What are the missing numbers in the pattern?</p> <p>1, 4, __, 64, __, 1,024</p>	<p>What are the missing numbers in the pattern?</p> <p>97, 86, __, 64, 53, __, 31</p>	<p>What are the missing numbers in the pattern?</p> <p>46, 37, __, 22, 16, __, 7</p>

Number Patterns Game Board

START

